Time Dependence of the Intensity of Diffracted Radiation Produced by a Relativistic Particle Passing through a Natural or Photonic Crystal

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Abstract

The formulas which describe the time evolution of radiation produced by a relativistic particle moving in a crystal are derived. It is shown that the conditions are realizable under which parametric (quasi-Cherenkov) radiation, transition radiation, diffracted radiation of the oscillator, surface quasi-Cherenkov and Smith-Purcell radiation last considerably longer than the time τ_p of the particle flight through the crystal. The results of carried out experiments demonstrate the presence of additional radiation peak appearing after the electron beam has left the photonic crystal.

Introduction

At present, the processes of diffracted radiation of photons by relativistic particles passing through crystals (natural or artificial spatially periodic structures) are intensively studied both theoretically and experimentally. Worthy of mention are such types of diffracted radiation as parametric (quasi-Cherenkov) radiation and diffracted radiation of a relativistic oscillator [1–3]. It should be noted, however, that until now, theoretical and experimental analysis of radiation produced by a relativistic particle passing through a crystal has focused on spectral-angular characteristics of radiation. Nevertheless, it was shown in [5, 6] that because of diffraction, photons produced through radiation in crystals have group velocity v_{gr}^{p}, which is appreciably smaller than the velocity v of a relativistic particle. As a result, the situation is possible in which radiation from the crystal still continues after the particle has passed through it [5, 6]. This enables studying time evolution of the process of photon radiation produced during the particle transmission through the crystal (natural or photonic), or during the particle flight along the surface of such crystals. In the present paper the formulas are derived, which describe the time evolution of radiation produced by a relativistic particle moving in a crystal. It is shown

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that the conditions are realizable under which parametric (quasi-Cherenkov) radiation, transition radiation, diffracted radiation of the oscillator, surface quasi-Cherenkov and Smith-Purcell radiation last considerably longer than the time τ_p of the particle flight through the crystal, i.e., much longer than $\tau_p \leq 10^{-9}$ s.

1 Spectral-angular distribution of radiation produced by a particle transmitted through a crystal

Let us first recall the conventional consideration of the radiation process in crystals [1, 8].

Both the spectral-angular density of radiation energy per unit solid angle $W_{\vec{n}\omega}$ and the differential number of emitted photons $dN_{\vec{n}\omega}\omega = 1/\hbar\omega \cdot W_{\vec{n}\omega}$ can be easily obtained if the field $\vec{E}(\vec{r},\omega)$ produced by a particle at a large distance $\vec{r}$ from the crystal is known [3]

$$W_{\vec{n}\omega} = \frac{er^2}{4\pi^2}\overline{\left|\vec{E}(\vec{r},\omega)\right|^2}, \tag{1}$$

The vinculum here means averaging over all possible states of the radiating system. In order to obtain $\vec{E}(\vec{r},\omega)$, Maxwell's equation describing the interaction of particles with the medium should be solved. The transverse solution can be found with the help of Green's function of this equation, which satisfies the expression:

$$G = G_0 + G_0\frac{\omega^2}{4\pi c^2}(\hat{\varepsilon} - 1)G, \tag{2}$$

G_0 is the transverse Green's function of Maxwell's equation at $\hat{\varepsilon} = 1$. It is given, for example, in [19].

Using G, we can find the field we are concerned with

$$E_n(\vec{r},\omega) = \int G_{nl}(\vec{r},\vec{r}',\omega)\frac{i\omega}{c^2}j_{0l}(\vec{r},\omega)d^3r', \tag{3}$$

where $n, l = x, y, z$, $j_{0l}(\vec{r},\omega)$ is the Fourier transformation of the e-th component of the current produced by a moving beam of charged particles (in the linear field approximation, the current is determined by the velocity and the trajectory of a particle, which are obtained from the equation of particle motion in the external field, by neglecting the influence of the radiation field on

the particle motion). Under the quantum-mechanical consideration the current j_0 should be considered as the current of transition of the particle-medium system from one state to another.

According to [3, 8], Green's function is expressed at $r \to \infty$ through the solution of homogeneous Maxwell's equations $E_n^{(-)}(\vec{r}, \omega)$ containing incoming spherical waves:

$$\lim_{r \to \infty} G_{nl}(\vec{r}, \vec{r}', \omega) = \frac{e^{ikr}}{r} \sum_S e_n^s E_{\vec{k}l}^{(-)s*}(\vec{r}', \omega), \tag{4}$$

where $\vec{e}^s$ is the unit polarization vector, $s - 1, 2$, $\vec{e}^1 \perp \vec{e}^2 \perp \vec{k}$.

If the electromagnetic wave is incident on a crystal of finite size, then at $r \to \infty$

$$\vec{E}_k^{(-)s}(\vec{r}, \omega) = \vec{e}^s e^{i\vec{k}\vec{r}} + \mathrm{const}\frac{e^{ikr}}{r},$$

and one can show that the relation between the solution $\vec{E}_k^{(-)s}$ and the solution of Maxwell's equation $\vec{E}^{(+)}(\vec{k}, \omega)$ describing scattering of a plane wave by the target (crystal), is given by:

$$\vec{E}_{\vec{k}}^{(-)s*} = \vec{E}_{-\vec{k}}^{(+)s} \tag{5}$$

Using (3), we obtain

$$E_n(\vec{r}, \omega) = \frac{e^{ikr}}{r} \frac{i\omega}{c^2} \sum_S e_n^s \int E_{\vec{k}}^{(-)s*}(\vec{r}, \omega) \vec{j}_0(\vec{r}', \omega) d^3r'. \tag{6}$$

As a result, the spectral energy density of photons with polarization s can be written in the form:

$$W_{\vec{n},\omega}^s = \frac{\omega^2}{4\pi^2 c^2} \overline{\left| \int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}, \omega) \vec{j}_0(\vec{r}, \omega) d^3r \right|^2}, \tag{7}$$

$$\vec{j}_0(\vec{r}, \omega) = \int e^{i\omega t} \vec{j}_0(\vec{r}, \omega) dt = eQ \int e^{i\omega t} \vec{v}(t) \delta(\vec{r} - \vec{r}(t)) dt, \tag{8}$$

where eQ is the charge of the particle, $\vec{v}(t)$ and $\vec{r}(t)$ are the velocity and the trajectory of the particle at moment t. By introducing (8) into (7) we get

$$dN_{\vec{n},\omega}^s = \frac{e^2 Q^2 \omega}{4\pi^2 \hbar c^3} \overline{\left| \int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}(t), \omega) \vec{v}(t) e^{i\omega t} d \right|^2} t. \tag{9}$$

Integration in (9) is carried out over the whole interval of the particle motion. It should be noted that the application of the solution of a homogeneous Maxwell's equation instead of the inhomogeneous one essentially simplifies the analysis of the radiation problem and enables one to consider various cases of radiation emission taking into account multiple scattering.

Using equations (7)–(9), one can easily obtain the explicit expression for the radiation intensity and that for the effect of multiple scattering on the process under study [3, 8, 9].

Consider, for example, the PXR radiation. Let a particle moving with a uniform velocity be incident on a crystal plate with the thickness L being $L \ll L_c$, where $L_c = (\omega q)^{-1/2}$ is the coherent length of bremsstrahlung $q = \overline{\theta}^2/4$ and $\overline{\theta}^2$ is the mean square angle of multiple scattering. The latter requirement allows neglecting the multiple scattering of particles by atoms. A theoretical method describing multiple scattering effect on the radiation process is given in [10].

According to (9), in order to determine the number of quanta emitted by a particle passing through the crystal plate, one should first find the explicit expressions for the solutions $\vec{E}_{\vec{k}}^{(-)s}$. As was mentioned above, the field $\vec{E}_{\vec{k}}^{(-)s}$ can be found from the relation $\vec{E}_{\vec{k}}^{(-)s} = (\vec{E}_{-\vec{k}}^{(+)s})^*$ if one knows the solution $\vec{E}_{\vec{k}}^{(+)s}$ describing the photon scattering by the crystal.

In the case of two strong waves excited under diffraction (the so-called two-beam diffraction case [11]), one can obtain the following set of equations for determining the wave amplitudes (see [12]):

$$\begin{aligned} &\left(\frac{k^2}{\omega^2} - 1 - \chi_0^*\right) \vec{E}_{\vec{k}}^{(-)s} c_s \chi_{-\vec{\tau}}^* \vec{E}_{\vec{k}_\tau}^{(-)s} = 0 \\ &\left(\frac{k^2}{\omega^2} - 1 - \chi_0^*\right) \vec{E}_{\vec{k}_\tau}^{(-)s} c_s \chi_{\vec{\tau}}^* \vec{E}_{\vec{k}}^{(-)s} = 0. \end{aligned} \tag{10}$$

Here $\vec{k}_{\vec{\tau}} = \vec{k} + \vec{\tau}$, $\vec{\tau}$ is the reciprocal lattice vector, χ_0, $\chi_{\vec{\tau}}$ are the Fourier components of the crystal susceptibility. It is well known that the crystal is described by a periodic susceptibility (see, for example, [11]:

$$\chi(\vec{r}) = \sum_{\vec{\tau}} \chi_{\vec{\tau}} \exp(i\vec{\tau}\vec{r}). \tag{11}$$

$c_s = \vec{e}^{\,s}\vec{e}^{\,s}_{\vec{\tau}}$, where $\vec{e}^{\,s}(\vec{e}^{\,s}_{\vec{\tau}})$ are the unit polarization vectors of the incident and diffracted waves, respectively.

The condition for the linear system (10) to be solvable leads to a dispersion equation that determines the possible wave vectors $\vec{k}$ in a crystal. These wave

vectors are convenient to present in the form:

$$\vec{k}_{\mu s} = \vec{k} + \vec{\kappa}^*_{\mu s}\vec{N}, \qquad \kappa^*_{\mu s} = \frac{\omega}{c\gamma_0}\varepsilon^*_{\mu s},$$

where $\mu = 1, 2$; $\vec{N}$ is the unit vector of a normal to the entrance crystal surface which is directed into the crystal,

$$\varepsilon_{1(2)s} = \frac{1}{4}\left[(1+\beta_1)\chi_0 - \beta_1\alpha_B\right] \pm \frac{1}{4}\left\{\left[(1-\beta_1)\chi_0 + \beta_1\alpha_B\right]^2 \right.$$
$$\left. +4\beta_1 C_s^2 \chi_{\vec{\tau}}\chi_{-\vec{\tau}}\right\}^{-1/2}. \tag{12}$$

$\alpha_B = (2\vec{k}\vec{\tau} + \tau^2)k^{-2}$ is the off-Bragg parameter ($\alpha_B = 0$ if the exact Bragg condition of diffraction is fulfilled),

$$\gamma_0 = \vec{n}_\gamma \cdot \vec{N}, \quad \vec{n}_\gamma = \frac{\vec{k}}{k}, \quad \beta_1 = \frac{\gamma_0}{\gamma_1}, \quad \gamma_1 = \vec{n}_{\gamma\tau} \cdot \vec{N}, \quad \vec{n}_{\gamma\tau} = \frac{\vec{k}+\vec{\tau}}{|\vec{k}+\vec{\tau}|}.$$

The general solution of (10) inside a crystal is:

$$\vec{E}^{(-)s}_{\vec{k}}(\vec{r}) = \sum_{\mu=1}^{2}\left[\vec{e}^{\,s} A_\mu \exp(i\vec{k}_{\mu s}\vec{r}) + \vec{e}^{\,s}_{\tau} A_{\tau\mu}\exp(i\vec{k}_{\mu s\tau}\vec{r})\right]. \tag{13}$$

Associating these solutions with the solutions of Maxwell's equations for the vacuum area, one can find the explicit form of $\vec{E}^{(-)s}_{\vec{k}}(\vec{r})$ throughout the space. It is possible to discriminate several types of diffraction geometries, namely, the Laue (a) and the Bragg (b) schemes are most well known.

(a) Let us consider the PXR in the Laue case.

In this case, the electromagnetic waves emitted by a particle in both the forward and the diffracted directions leave the crystal through the same surface ($k_z > 0, k_z + \tau_z > 0$), the z-axis is parallel to the normal N (where N is the normal to the crystal surface being directed inside a crystal). By matching the solutions of Maxwell's equations on the crystal surfaces with the help of (10), (12), (13), one can obtain the following expressions for the Laue case:

$$\vec{E}^{(-)s}_{\vec{k}} = \left\{\vec{e}^{\,s}\left[-\sum_{\mu=1}^{2}\xi^{0*}_{\mu s} e^{-i\frac{\omega}{\gamma_0}\varepsilon^*_{\mu s}L}\right] e^{i\vec{k}\vec{r}} + e^s_{\vec{\tau}}\beta_1\left[\sum_{\mu=1}^{2}\xi^{\tau*}_{\mu s}e^{-i\frac{\omega}{\gamma_0}\varepsilon^*_{\mu s}L}\right]e^{i\vec{k}_\tau\vec{r}}\right\}\theta(-z)$$
$$+\left\{\vec{e}^{\,s}\left[-\sum_{\mu=1}^{2}\xi^{0*}_{\mu s}e^{-i\frac{\omega}{\gamma_0}\varepsilon^*_{\mu s}(L-z)}\right]e^{i\vec{k}\vec{r}} + e^s_{\vec{\tau}}\beta_1\left[\sum_{\mu=1}^{2}\xi^{\tau*}_{\mu s}e^{-i\frac{\omega}{\gamma_0}\varepsilon^*_{\mu s}(L-z)}\right]e^{i\vec{k}_\tau\vec{r}}\right\}$$

$$\times\theta(L-z)\theta(z)+\vec{e}^{s}e^{i\vec{k}\vec{r}}\theta(z-L), \tag{14}$$

where

$$\xi^{0}_{1,2s}=\mp\frac{2\varepsilon_{2,1s}-\chi_0}{2(\varepsilon_{2s}-\varepsilon_{1s})};$$
$$\xi^{\tau}_{1,2s}=\mp\frac{c_s\chi_{-\tau}}{2(\varepsilon_{2s}-\varepsilon_{1s})};$$
$$\theta(z)=\begin{cases}1, & \text{if } z\geq 0\\ 0, & \text{if } z<0.\end{cases}$$

Substitution of (14) into (9) gives for the Laue case the differential number of quanta of the forward directed parametric X-rays with the polarization vector $\vec{e}_s$:

$$\frac{d^2N^L_{0s}}{d\omega d\Omega}=\frac{e^2Q^2\omega}{4\pi^2\hbar c^3}(\vec{e}^{s}\vec{v})^2\left|\sum_{\mu=1,2}\xi^0_{\mu s}e^{i\frac{\omega}{c\gamma_0}\varepsilon_{\mu s}L}\left[\frac{1}{\omega-\vec{k}\vec{v}}-\frac{1}{\omega-\vec{k}^{*}_{\mu s}\vec{v}}\right]\right.$$
$$\left.\times\left[e^{i(\omega-\vec{k}^{*}_{\mu s}\vec{v})T}-1\right]\right|^2, \tag{15}$$

where $T=L/c\gamma_0$ is the particle time of flight; $\vec{e}_1\parallel[\vec{k}\vec{\tau}]$; $\vec{e}_2\parallel[\vec{k}\vec{e}_1]$.

One can see that formula (15) looks like the formula which describes the spectral and angular distribution of the Cherenkov and transition radiations in the matter with the index of refraction $n_{\mu s}=k_{z\mu s}/k_z=1+\kappa_{\mu s}/k_z$.

The spectral angular distribution for photons in the diffraction direction $\vec{k}_\tau=\vec{k}+\vec{\tau}$ can be obtained from (15) by a simple substitution

$$\vec{e}_s\rightarrow\vec{e}_{s\tau},\qquad \xi^0_{\mu s}\rightarrow\beta_1\xi^{\tau}_{\mu s},$$
$$\xi^{\tau}_{1(2)s}=\pm\frac{\chi_\tau c_s}{2(\varepsilon_{1s}-\varepsilon_{2s})}$$
$$\vec{k}\rightarrow\vec{k}_\tau,\quad \vec{k}_{\mu s}\rightarrow\vec{k}_{\tau\mu s}=\vec{k}_{\mu s}+\tau.$$

(b) Now let us consider PXR in the Bragg case. In this case, side by side with the electromagnetic wave emitted in the forward direction, the electromagnetic wave emitted by a charged particle in the diffracted direction and leaving the crystal through the surface of the particle entrance can be observed. By matching the solutions of Maxwell's equations on the crystal surface with the help of (10), (12), (13), one can get the formulas for the Bragg diffraction schemes.

It is interesting that the spectral angular distribution for photons emitted in the forward direction can be obtained from (15) by the following substitution, $\xi^0_{\mu s} \rightarrow \gamma_{\mu s}$,

$$\gamma^0_{1(2)s} = \frac{2\varepsilon_{2(1)s} - \chi_0}{(2\varepsilon_{2(1)s} - \chi_0) - (2\varepsilon_{1(2)s} - \chi_0) e^{i\frac{\omega}{\gamma_0}(\varepsilon_{2(1)s} - \varepsilon_{1(2)s})L}} \tag{16}$$

The spectral angular distribution of photons emitted in the diffracted direction can be obtained from (15) by substitution

$$\vec{e}_s \rightarrow \vec{e}_{s\tau}, \quad \vec{k} \rightarrow \vec{k}_\tau, \quad k_{\mu s} \rightarrow \vec{k}_{\mu\tau s}, \quad \xi^0_{\mu s} e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s} L} \rightarrow \gamma^\tau_{\mu s},$$

where

$$\gamma^\tau_{1(2)s} = -\frac{\beta_1 c_s \chi_\tau}{(2\varepsilon_{2(1)s} - \chi_0) - (2\varepsilon_{1(2)s} - \chi_0) e^{i\frac{\omega}{\gamma_0}(\varepsilon_{2(1)s} - \varepsilon_{1(2)s})L}}. \tag{17}$$

Let us note that the above formulas fully describe parametric (quasi-Cherenkov) radiation in natural and photonic crystals and they certainly include that contribution to radiation, which goes over to ordinary transition radiation, if the radiation is considered outside the region of diffraction reflection. A description of diffracted radiation of a relativistic oscillator is given in [1, 2] and the reference therein.

Let us take notice of the fact that in photonic crystals built from metal threads with the diameter smaller than or comparable with λ, the value of $\chi(\tau)$ is practically independent on τ. As a result, it is possible to effectively excite radiation in, e.g., the terahetrz range in a lattice with a period of several millimeters.

When a particle travels in a vacuum near the surface of a spatially periodic medium, new kinds of radiation arise [13, 14] – surface parametric (quasi-Cherenkov) X-ray radiation (SPXR) and surface DRO (see Figure 1). This phenomenon takes place under the condition of uncoplanar surface diffraction, first considered in [15].

The solution of Maxwell's equation $\vec{E}^{(+)}_{\vec{k}}(\vec{r})$ in this case of uncoplanar surface diffraction was obtained in [15]. It was shown that the surface diffraction in the two-wave case is characterized by two angles of total reflection (several angles in the case of multi-wave diffraction [16]). The solution obtained in [16] contains the component, which describes the state that damps with growing distance from the surface of the medium, both within the material and in the vacuum, and which describes a surface wave, i.e., a wave in which the energy flux is directed along the boundary of the surface of a spatially periodic target

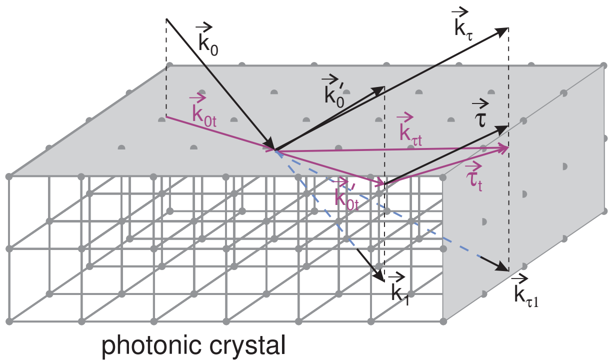

Figure 1. Surface diffraction of a radiated photon

(see review [17]). According to [15], this solution, which describes scattering of a plane wave by the target under the surface diffraction geometry, can be written in the form:

$$\vec{E}_{\vec{k}}^{(+)s} = e_s e^{i\vec{k}\vec{r}} + A_s(\vec{k},\omega)e^{i\vec{k}_1\vec{r}} + B_s(\vec{k},\omega)e^{i\vec{k}_2\vec{r}}, \tag{18}$$

where the wave vector in a vacuum $\vec{k} = (\vec{k}_t, \vec{k}_\perp)$, $\vec{k}_1 = (\vec{k}_t, -\vec{k}_\perp)$, $\vec{k}_2 = (\vec{k}_{2t}, -\vec{k}_{2\perp})$, $|\vec{k}_{2\perp}| = \sqrt{k^2 - k_{2t}^2}$, $\vec{k}_{2t} = \vec{k}_t + \vec{\tau}$, $\vec{k}_t$ is the component of the wave vector that is parallel to the surface, $\vec{\tau}$ is the reciprocal lattice vector, ω is the photon frequency. The amplitudes A_s and B_s are given in [9, 14]. Substituting the solution $\vec{E}_{\vec{k}}^{(-)s} = (\vec{E}_{-\vec{k}}^{(+)s})^*$ into (3), we can find the spectral-angular distribution of SPXR and DRO.

2 Time dependence of the intensity of radiation produced by a particle transmitted through a crystal

The intensity $I(t)$ of radiation produced by a particle which has passed through a crystal can be found with known intensity of the electric field $\vec{E}(\vec{r},t)$) (magnetic field $\vec{H}(\vec{r},t)$) of the electromagnetic wave, which is produced by this particle [18],

$$I(t) = \frac{c}{4\pi}|\vec{E}(\vec{r},t)|^2 r^2 d\Omega, \tag{19}$$

where r is the distance from the crystal, which is assumed to be larger than the crystal size.

The field $\vec{E}(\vec{r},t)$ can be presented as an expansion in a Fourier series

$$\vec{E}(\vec{r},t) = \frac{1}{2\pi}\int \vec{E}(\vec{r},\omega)e^{-i\omega t}d\omega. \tag{20}$$

According to the results obtained in [3,7,9], at a long distance from the crystal, the Fourier component can be written as follows:

$$\vec{E}(\vec{r},t) = \frac{e^{ikr}}{r}\frac{i\omega}{c^2}\sum_s e_i^s \int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}'\omega)\vec{j}(\vec{r}',\omega)d^3r'. \tag{21}$$

where $i = 1,2,3$ (and correspond(s) to the coordinate axes x, y, z), e_i^s is the i-component of the wave polarization vector $\vec{e}^s$; $s = 1,2$; $\vec{E}_{\vec{k}}^{(-)s}$ is the solution of Maxwell's equations describing scattering of a plane wave with a wave vector $\vec{k} = k\frac{\vec{r}}{r}$ and the asymptotic of a converging spherical wave,

$$\vec{j}(\vec{r},\omega) = \int \vec{j}(\vec{r},t)e^{i\omega t}dt \tag{22}$$

$\vec{j}(\vec{r},\omega) = Q\vec{v}(t)\delta(\vec{r}-\vec{r}(t))$ is the is the current density of the particle with charge Q, $\vec{r}(t)$ is the particle coordinate at time t.

The explicit form of the expressions $\vec{E}^{(-)s}$ describing diffraction of the electromagnetic wave in a crystal in the Laue and Bragg cases is given in [3,8,12] (See Section 1).

Now let us take a closer look at the expression for the amplitude $A(\omega)$ of the emitted wave:

$$A_{\vec{k}}^s(\omega) = \frac{i\omega}{c^2}\int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}',\omega)\vec{j}(\vec{r}',\omega)d^3r'. \tag{23}$$

Using (22), (23) can be recast as follows

$$\begin{aligned} A_{\vec{k}}^s(\omega) &= \frac{i\omega}{c^2}\int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}',\omega)Q\vec{v}(t)\delta(\vec{r}'-\vec{r}(t))e^{i\omega t}dtd^3r' \\ &= \frac{i\omega Q}{c^2}\int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}(t),\omega)\vec{v}(t)e^{i\omega t}dt \end{aligned} \tag{24}$$

Recall that $\vec{E}_{\vec{k}}^{(-)s*} = \vec{E}_{-\vec{k}}^{(+)s}$, where the field $\vec{E}_{-\vec{k}}^{(+)s}$ is the solution of Maxwell's equations describing scattering by a crystal of a plane wave with wave vector $(-\vec{k})$ and the asymptotics of a diverging wave at infinity. According to (24), the radiation amplitude is determined by the field $\vec{E}_{\vec{k}}^{(-)s}$ taken at point $\vec{r}(t)$ of particle location at time t and integrated over the time of particle motion.

Let us consider in more detail the constant motion of a particle in passing through the crystal. In this case, parametric quasi-Cherenkov radiation can appear [1,8], which includes, as a particular case, diffracted transition radiation. The explicit formulas for the radiation amplitude in the case of two-wave

diffraction of photons in crystals for the Laue and Bragg geometries are given in [3, 8, 12] (Section 1).

From (20), (21), and (23) follows that the expression for the electromagnetic wave emitted by the particle passing through the crystal (natural or photonic) can be presented in a form:

$$\vec{E}_i(\vec{r},t) = \frac{1}{2\pi r}\sum_s e_i^s \int A^s_{\vec{k}}(\omega) e^{-i\omega(t-\frac{r}{c})} d\omega, \tag{25}$$

i.e., $\vec{E}_i(\vec{r},t) = \frac{1}{r}\sum\limits_s e_i^s A^s_{\vec{k}}(t-\frac{r}{c})$.

From (25) follows that the time dependence of the form of the pulse $I(\vec{r},t)(\vec{E}(\vec{r},t))$ of radiation generated by a particle passing through the crystal is determined by the dependence of the radiation amplitude $A^s_{\vec{k}}(\omega)$ on frequency. According to the explicit expression for the radiation amplitudes given in [3, 8, 12], the radiation amplitudes $A^s_{\vec{k}}(\omega)$ can be presented as sums proportional to the amplitudes of diffraction reflection from the crystal and to the amplitude of wave transmission through the crystal. For example, for the case of forward parametric radiation in the Laue geometry

$$\begin{aligned} A^s_{\vec{k}}(\omega) = {} & \frac{Q}{c^2}(\vec{e}^{s}\vec{v}) \sum_{\mu=1,2} \xi^0_{\mu s} e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s} L} \\ & \times \left[\frac{1}{\omega-\vec{k}\vec{v}} - \frac{1}{\omega-(\vec{k}+\kappa_{\mu s}\vec{N})\vec{v}}\right]\left[e^{i(\omega-(\vec{k}+\kappa_{\mu s}\vec{N})\vec{u})\frac{L}{c\gamma_0}} - 1\right] \end{aligned} \tag{26}$$

Thus, the time dependence of the from of the radiation pulse is determined by the time dependence of the radiation amplitude $A^s_{\vec{k}}(t-\frac{r}{c})$.

By way of example, let us consider the characteristics of the time dependence of radiation produced by a particle passing through the crystal for a wave packet passing through the crystal [2, 5, 6]

Let us consider the pulse of electromagnetic radiation passing through the medium with the index of refraction $n(\omega)$. The group velocity of the wave packet is as follows:

$$v_{gr} = \left(\frac{\partial \omega n(\omega)}{c\partial\omega}\right)^{-1} = \frac{c}{n(\omega)+\omega\frac{\partial n(\omega)}{\partial\omega}}, \tag{27}$$

where c is the speed of light, ω is the quantum frequency.

In the X-ray range ($\sim$tens of keV) the index of refraction has the universal form $n(\omega) = 1 - \frac{\omega_L^2}{2\omega^2}$, ω_L is the Langmuir frequency. Additionally, $n - 1 \simeq 10^{-6} \ll 1$. Substituting $n(\omega)$ into (27), one can obtain that $v_{gr} \simeq c\left(1 - \frac{\omega_L^2}{\omega^2}\right)$. It is clear that the group velocity is close to the speed of light. Therefore the time delay of the wave packet in a medium is much shorter than the time needed for passing the path equal to the target thickness in a vacuum.

$$\Delta T = \frac{l}{v_{gr}} - \frac{l}{c} \simeq \frac{l}{c}\frac{\omega_L^2}{\omega^2} \ll \frac{l}{c}. \tag{28}$$

To consider the pulse diffraction in a crystal, one should solve Maxwell's equations that describe a pulse passing through a crystal. Maxwell's equations are linear, therefore it is convenient to use the Fourier transform in time and to rewrite these equations as functions of frequency:

$$\left[-curl\ curl\ \vec{E}_{\vec{k}}(\vec{r},\omega) + \frac{\omega^2}{c^2}\vec{E}_{\vec{k}}(\vec{r},\omega)\right]_i + \chi_{ij}(\vec{r},\omega)\ E_{\vec{k},j}(\vec{r},\omega) = 0, \tag{29}$$

where $\chi_{ij}(\vec{r},\omega)$ is the spatially periodic tensor of susceptibility; $i, j = 1, 2, 3$ repeated indices imply summation.

Making the Fourier transformation of these equations in coordinate variables, one can derive a set of equations associating the incident and diffracted waves. When two strong waves are excited under diffraction (the so-called two-beam diffraction case), the following set of equations for determining the wave amplitudes can be obtained:

$$\begin{aligned} &\left(\frac{k^2}{\omega^2} - 1 - \chi_0\right)\vec{E}^s_{\vec{k}} - c_s\chi_{-\vec{\tau}}\vec{E}^s_{\vec{k}_\tau} = 0 \\ &\left(\frac{k_\tau^2}{\omega^2} - 1 - \chi_0\right)\vec{E}^s_{\vec{k}_\tau} - c_s\chi_{\vec{\tau}}\vec{E}^s_{\vec{k}} = 0 \end{aligned} \tag{30}$$

Here $\vec{k}$ is the wave vector of the incident wave, $\vec{k}_{\vec{\tau}} = \vec{k} + \vec{\tau}$, $\vec{\tau}$ is the reciprocal lattice vector; $\chi_0, \chi_{\vec{\tau}}$ are the Fourier components of the crystal susceptibility:

$$\chi(\vec{r}) = \sum_{\vec{\tau}} \chi_{\vec{\tau}} \exp(i\vec{\tau}\vec{r}) \tag{31}$$

$C_s = \vec{e}^{\,s}\vec{e}^{\,s}_{\vec{\tau}}$, $\vec{e}^{\,s}(\vec{e}^{\,s}_{\vec{\tau}})$ are the unit polarization vectors of the incident and diffracted waves, respectively.

The solvability condition for the linear system (30) leads to a dispersion equation that determines the possible wave vectors $\vec{k}$ in a crystal. It is convenient

to present these wave vectors as:

$$\vec{k}_{\mu s} = \vec{k} + æ_{\mu s}\vec{N},\ æ_{\mu s} = \frac{\omega}{c\gamma_0}\ \varepsilon_{\mu s},$$

where $\mu = 1, 2$; $\vec{N}$ is the unit vector of a normal to the entrance surface of the crystal, which is directed into the crystal,

$$\varepsilon_s^{(1,2)} = \frac{1}{4}[(1+\beta)\chi_0 - \beta\alpha_B] \pm \frac{1}{4}\left\{[(1+\beta)\chi_0 - \beta\alpha_B - 2\chi_0]^2 + 4\beta C_S^2 \chi_{\vec{\tau}}\chi_{-\vec{\tau}}\right\}^{1/2}, \quad (32)$$

$\alpha_B = (2\vec{k}\vec{\tau} + \tau^2)k^{-2}$ is the off-Bragg parameter ($\alpha_B = 0$ when the Bragg condition of diffraction is exactly fulfilled),

$$\gamma_0 = \vec{n}_\gamma \cdot \vec{N}, \quad \vec{n}_\gamma = \frac{\vec{k}}{k}, \quad \beta = \frac{\gamma_0}{\gamma_1}, \quad \gamma_1 = \vec{n}_{\gamma\tau} \cdot \vec{N}, \quad \vec{n}_{\gamma\tau} = \frac{\vec{k} + \vec{\tau}}{|\vec{k} + \vec{\tau}|}$$

The general solution of equations (29), (30) inside a crystal is:

$$\vec{E}^s_{\vec{k}}(\vec{r}) = \sum_{\mu=1}^{2}\left[\vec{e}^{\ s} A_\mu \exp(i\vec{k}_{\mu s}\vec{r}) + \vec{e}_\tau^{\ s} A_{\tau\mu} \exp(i\vec{k}_{\mu s\tau}\vec{r})\right] \quad (33)$$

Associating these solutions with the solutions of Maxwell's equation for the vacuum area one can find the explicit expression for $\vec{E}^s_{\vec{k}}(\vec{r})$ throughout the space. It is possible to discriminate several types of diffraction geometries, namely, the Laue and the Bragg schemes, which are most well-known [22].

In the case of two-wave dynamical diffraction, the crystal can be described by two effective indices of refraction

$$n_s^{(1,2)} = 1 + \varepsilon_s^{(1,2)},$$

$$\varepsilon_s^{(1,2)} = \frac{1}{4}\left\{\chi_0(1+\beta) - \beta\alpha \pm \sqrt{(\chi_0(1-\beta) + \beta\alpha)^2 + 4\beta C_s \chi_\tau \chi_{-\tau}}\right\}. \quad (34)$$

The diffraction is significant in the narrow range near the Bragg frequency, therefore χ_0 and χ_τ can be considered as constants and the dependence on ω should be taken into account for $\alpha = \frac{2\pi\overrightarrow{\tau}(2\pi\overrightarrow{\tau}+2\overrightarrow{k})}{k^2} = -\frac{(2\pi\tau)^2}{k_B^3 c}(\omega - \omega_B)$, where $k = \frac{\omega}{c}$; $2\pi\overrightarrow{\tau}$ is the reciprocal lattice vector which characterizes the set of planes where the diffraction occurs; Bragg frequency is determined by the condition $\alpha = 0$.

From (27), (34) one can obtain

$$v_{gr}^{(1,2)s}=\frac{c}{n^{(1,2)}(\omega)\pm\beta\frac{(2\pi\tau)^2}{4k_B^2}\frac{(\chi_0(1-\beta)+\beta\alpha)}{\sqrt{(\chi_0(1-\beta)+\beta\alpha)^2+4\beta C_s\chi_\tau\chi_{-\tau}}}}. \tag{35}$$

In the general case $(\chi_0(1-\beta)+\beta\alpha)\simeq 2\sqrt{\beta}\chi_0$, therefore the term that is added to $n_s^{(1,2)}(\omega)$ in the denominator (35) is of the order of 1. Moreover, v_{gr} significantly differs from c for the antisymmetric diffraction ($|\beta|\gg 1$). It should be noted that because of the complicated character of the wave field in a crystal, one of $v_{gr}^{(i)s}$ can appear to be much higher than c and negative. When β is negative the radicand in (35) can become zero (Bragg reflection threshold) and $v_{gr}\to 0$. It should be noted that in the presence of a variable external field, a crystal can be described by the effective indices of refraction which depend on the external field frequency Ω . Therefore in this case v_{gr} appears to be the function of Ω . This can be easily observed in the conditions of X-ray-acoustic resonance. The performed analysis allows one to conclude that the center of the X-ray pulse in a crystal can undergo a significant delay $\Delta T\gg\frac{l}{c}$ available for experimental investigation. Thus, when $\beta=10^3$, $l=0.1$ cm and $l/c\simeq 3\cdot 10^{-12}$, the delay time can be estimated as $\Delta T\simeq 3\cdot 10^{-9}$sec.

Let us study now the time dependence of the delay law of radiation after passing through a crystal. Assuming that $B(\omega)$ is the reflection or transmission amplitude coefficients of a crystal, one can obtain the following expression for the pulse form

$$E(t)=\frac{1}{2\pi}\int B(\omega)E_0(\omega)e^{-i\omega t}d\omega=\int B(t-t')E_0(t')dt'. \tag{36}$$

where $E_0(\omega)$ is the amplitude of the electromagnetic wave incident on a crystal

In accordance with the general theory, for the Bragg geometry, the amplitude of the diffraction-reflected wave for the crystal width much greater than the absorbtion length can be written as [22]:

$$B_s(\omega)= \tag{37}$$
$$-\frac{1}{2\chi_\tau}\left\{\chi_0(1+|\beta|)-|\beta|\,\alpha-\sqrt{(\chi_0(1-|\beta|)-|\beta|\,\alpha)^2-4\,|\beta|\,C_s\chi_\tau\chi_{-\tau}}\right\}$$

In the absence of resonance scattering, the parameters χ_0 and $\chi_{\pm\tau}$ can be considered as constants and frequency dependence is defined by the term

$\alpha=-\frac{(2\pi\tau)^2}{k_B^3 c}(\omega-\omega_B)$. So, $B_s(t)$ can be found from

$$B_s(t)=-\frac{1}{4\pi\chi_\tau} \qquad (38)$$
$$\times\int\left\{\chi_0(1+|\beta|)-|\beta|\,\alpha-\sqrt{(\chi_0(1-|\beta|)-|\beta|\,\alpha)^2-4\,|\beta|\,C_s\chi_\tau\chi_{-\tau}}\right\}e^{-i\omega t}d\omega.$$

The Fourier transform of the first term results in $\delta(t)$ and we can neglect it because the delay is described by the second term. The second term can be calculated by the methods of the theory of function of complex argument:

$$B_s(t)=-\frac{i}{4\chi_\tau}\,|\beta|\,\frac{(2\pi\tau)^2}{k_B^2\omega_B}\frac{J_1(a_s t)}{t}e^{-i(\omega_B+\Delta\omega_B)t}\theta(t), \qquad (39)$$

or

$$B_s(t)=-\frac{i\sqrt{|\beta|}}{2}\frac{J_1(a_s t)}{a_s t}e^{-i(\omega_B+\Delta\omega_B)t}\theta(t), \qquad (40)$$

where

$$a_s=\frac{2\sqrt{C_s\chi_\tau\chi_{-\tau}}\omega_B}{\sqrt{|\beta|\frac{(2\pi\tau)^2}{k_B^2}}},\ \Delta\omega_B=-\frac{\chi_0(1+|\beta|)\omega_B k_B^2}{|\beta|\,(2\pi\tau)^2}.$$

Since χ_0 and χ_τ are complex, both a_s and $\Delta\omega_B$ have real and imaginary parts. According to (39)–(40), in the case of Bragg reflection of a short pulse (the pulse frequency bandwidth $\gg$ frequency bandwidth of the total reflection range) both the instantly reflected pulse and the pulse with amplitude undergoing damped beatings appear. Beatings period increases with $|\beta|$ grows and χ_τ decrease. Pulse intensity can be written as

$$I_s(t)\sim|B_s(t)|^2=\frac{|\beta|}{2}\left|\frac{J_1(a_s t)}{at}\right|^2 e^{-2\mathtt{Im}\Delta\omega_B t}\theta(t). \qquad (41)$$

It is evident that the reflected pulse intensity depends on the orientation of photon polarization vector $\vec{e}_s$ and undergoes the damping oscillations on time.

Let us evaluate the effect. Characteristic values are $\mathtt{Im}\Delta\omega_B\sim\mathtt{Im}\chi_0\omega_B$ and $\mathtt{Im}a\sim\frac{\mathtt{Im}\chi_\tau\omega_B}{\sqrt{\beta}}$. For 10 keV for the crystal of Si $\mathtt{Im}\chi_0=1,6\cdot10^{-7}$, for LiH

$\mathtt{Im}\chi_0 = 7,6 \cdot 10^{-11}, \mathtt{Im}\chi_\tau = 7 \cdot 10^{-11}$, for LiF $\mathtt{Im}\chi_0 \sim 10^{-8}$. Consequently, the characteristic time τ for the exponent decay in (41) can be estimated as follows ($\omega_B = 10^{19}$):

for Si the characteristic time $\tau \sim 10^{-12}$ sec, for LiF the characteristic time $\tau \sim 10^{-10}$ sec, for LiH the characteristic time $\tau \sim 10^{-9}$ sec!!

The reflected pulse also undergoes oscillations, the period of which increases with growing $|\beta|$ and decreasing $\mathtt{Re}\chi_\tau$. This period can be estimated for $\beta = 10^2$ and $\mathtt{Re}\chi_\tau \sim 10^{-6}$ as $T \sim 10^{-12}$ sec (for Si, LiH, LiF).

When the resolving time of the detecting equipment is greater than the oscillation period, the expression (41) should be averaged over the period of oscillations. Then, for the time intervals when $\mathtt{Re}a_s t \gg 1$, $\mathtt{Im}\Delta\omega_B t \ll 1$ the delay law (41) has the power function form:

$$I_s(t) \sim t^{-3}.$$

In the case of multi-wave diffraction, the time delay for the photon exit from the crystal will be even more appreciable.

For an artificial spatially periodic medium (diffraction grating, photonic crystal), the parameter $g_{0\,1}$ can vary over a wide range. For example, according to [23], for a photonic crystal built from tungsten threads of $100\mu m$ in diameter, the parameter $g_{0\,1} \sim \frac{1}{\omega^2}$ has the value of $g_{0\,1} \sim 10^{-2}$ in a 10 GHz range. As a result, in this range we have T (10 GHz)$\sim \frac{\sqrt{\beta}}{|g_{0\,1}|\omega_B} \sim \sqrt{\beta} \cdot 10^{-9}$. At the same time, in the terahertz range, T (1 THz) due to the drop of $g_{0\,1}$ (T increases proportionally to ω, the parameter a decreases: $a \sim \frac{1}{\omega_B}$), we have the period T (1 THz) $\sim \sqrt{\beta} \cdot 10^{-6}$. As is seen, the oscillations of radiation from photonic crystals are quite observable.

So the time $\tau_{ph} = \frac{L}{v_{gr}}$ that the photon spends in the crystal can be longer than the flight time $\tau_p = \frac{L}{v}$ of a relativistic particle in a crystal. Hence, the emission of diffraction-related radiation (quasi-Cherenkov, transition, diffracted radiation of an oscillator, surface parametric radiation and others) produced by a relativistic particle will continue after the particle has left the crystal (see Fig.2) Under diffraction conditions, the crystal acts as a high-quality resonator [1, 24].

It should be noted, of course, that in observation of oscillations, one should either register the moment of particle entrance into the crystal or use a short bunch of particles with duration much shorter than the oscillation period. In the X-ray range, such situation is typical of electron buches, which are applied for creating X-ray FELs (DESY). (The bunch duration in such FELs is

tens-hundreds of femptoseconds). In the terahertz range, much longer bunches are required, so there are not serious experimental problems in this case. If the bunch duration is large in comparison with the duration of the radiation pulse or the time of the electron entrance into the crystal is not registered, which occurs in a conventional experimental arrangement, then the intensity $I(t)$ should be integrated over longer observation time intervals. As a result, we, in fact, obtain the expression (1) integrated over all frequencies, i.e., an ordinary stationary angular distribution of radiation. If the response time of the devices detecting τ_D (or the flight time of the particle in a crystal, or the bunch duration) is comparable with the oscillation period, then $I(t)$ should be integrated over the interval τ_D. In this case oscillations will disappear, but we will observe the power-law decrease in the intensity of radiation from the crystal.

In according with the above analysis some experiments are carried to observe delay of radiation pulse in a photonic crystal used for VFEL lasing [25–28]. In these experiments the additional radiation peak (see Fig.2) is observed at studies of lasing of VFEL with "grid" photonic crystals in backward wave oscillator regime. This peak appears when the electron beam has left the resonator.

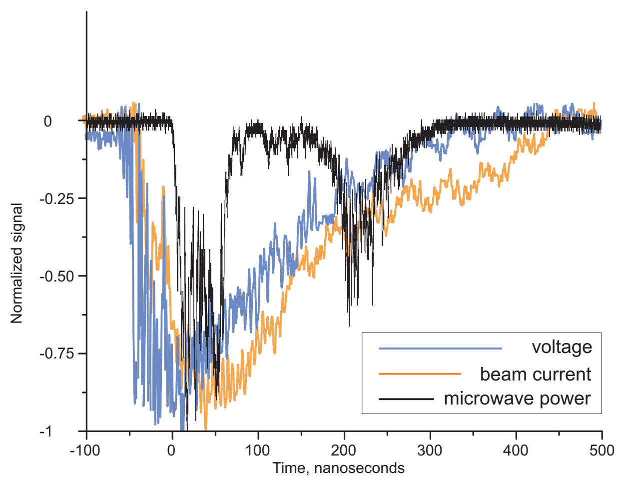

Figure 2. Detected microwave signal (black curve) synchronized with the beam current and electron gun voltage

It should be mentioned here that backward wave oscillator regime implies generation in presence of Bragg diffraction, therefore, under some conditions the group velocity could appear even to be close to 0 (see equation (35)). The observed delay (Fig.2) corresponds to $v_{gr} \sim 10^8$ cm/s, i.e. $\frac{v_{gr}}{c} \sim 10^{-2}$.

In travelling wave regime, which corresponds to case of Laue diffraction, such long delay can not be obtained (according to (35) for $\beta > 0$ the group velocity v_{gr} changes insignificantly). Particularly, in our experiments with Cherenkov generator without diffraction grating no additional peaks are detected, because

the group velocity in this case changes insignificantly due to the same reasons as in the Laue case.

And after all note that diffraction of a pulse of radiation produced by an external radiation source in a periodic structure could be accompanied by appearance of several transmitted or reflected radiation pulses (pulses of photons) (see [29]).

Conclusion

The formulas which describe the time evolution of radiation produced by a relativistic particle moving in a crystal are derived. It is shown that the conditions are realizable under which parametric (quasi-Cherenkov) radiation, transition radiation, diffracted radiation of the oscillator, surface quasi-Cherenkov and Smith-Purcell radiation last considerably longer than the time τ_p of the particle flight through the crystal. The results of carried out experiments demonstrate the presence of additional radiation peak appearing after the electron beam has left the photonic crystal.

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